SIMPLE SOLUTIONS FOR MODELLING THE NON-UNIFORM SUBSTRATE DOPING

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ABSTRACT

Present-day CMOS processes use ion implantation to adjust the threshold voltages and to avoid the punchthrough effect. The substrate doping in the transistor can thus no longer be considered as uniform, and accurate modelling of this effect is required for precise analog circuit simulation. Two new solutions taking this effect into account are proposed and compared with other existing approaches. The results of these new models are compared with 2-D device simulations and measurements of a 2 µm CMOS low voltage process.

1. INTRODUCTION

The non-uniform doping mainly affects the relation between the depletion charge (per unit area) Q_B' and the channel voltage V_{ch} in strong inversion. This is best illustrated by considering the variation of this charge as a function of the channel voltage compared to its value in an equilibrium situation where $V_{ch} = 0$. This variation can also be expressed in terms of the threshold variation ΔV_{TB}:

$$\Delta V_{TB} \equiv V_{TB}(V_{ch}) - V_{ch} - V_{T0} = \left|\frac{Q'_B}{C'_{ox}}\right|_{V_{ch}>0} - \left|\frac{Q'_B}{C'_{ox}}\right|_{V_{ch}=0} \quad (1)$$

where the threshold voltage V_{TB} referenced to the substrate has been used, since in the EKV model all voltages are referenced to the bulk [1]. It can of course also be expressed as a variation of the more conventional threshold voltage referenced to the source V_{TH}:

$$\Delta V_{TB} = V_{TH}(V_{ch}) - V_{T0} = \left|\frac{Q'_B}{C'_{ox}}\right|_{V_{ch}>0} - \left|\frac{Q'_B}{C'_{ox}}\right|_{V_{ch}=0} \quad (2)$$

V_{T0} is the threshold voltage corresponding to the gate voltage V_G for which the inversion charge (Q'_{inv}) forming the channel is zero in an equilibrium situation ($V_{ch} = 0$). The EKV model uses the so called pinch-off voltage V_P to take into account the substrate effect. It is defined as the particular value of the channel voltage V_{ch} for which the inversion charge Q'_{inv} becomes zero at a given voltage V_G larger than V_{T0}. The relation between V_G and V_P assuming a uniform doping is given by:

$$V_P = V_G - V_{T0} - \gamma \cdot \left[\sqrt{V_G - V_{T0} + \left(\sqrt{\Psi_0} + \frac{\gamma}{2}\right)^2} - \left(\sqrt{\Psi_0} + \frac{\gamma}{2}\right)\right] \quad (3)$$

where Ψ_0 is the approximation of the surface potential in strong inversion (and for $V_{ch} = 0$) which is approximately given by:

$$\Psi_0 = 2\Phi_F + \text{several } U_T = 2 \cdot U_T \cdot \ln(N_s/n_i) + \text{several } U_T \quad (4)$$

The important factor γ is the substrate factor which is a function of the doping N_s, and is given by:

$$\gamma \equiv \gamma(N_s) = \frac{\sqrt{2 \cdot \varepsilon_0 \cdot \varepsilon_{si} \cdot N_s}}{C_{ox}} \quad (5)$$

This voltage V_P can be seen as the equivalent channel voltage to apply in order to cancel the effect due to V_G. It is a way to refer the effect of the gate voltage directly to the channel. The slope of the V_G versus V_P characteristic corresponds to the slope factor n, which is related to the slope of the I_D versus V_{GB} characteristic in weak inversion (in a log-lin scale). The slope factor n is a function of V_P and is given for a uniform doping by:

$$n \equiv \frac{dV_G}{dV_P} = 1 + \frac{\gamma}{2 \cdot \sqrt{\Psi_0 + V_P}} \quad (6)$$

The non-uniform doping will obviously also affect the relation existing between the pinch-off voltage and the gate voltage, mainly through the substrate factor γ and of course also the slope factor n. This effect can be modelled by changing the V_P versus V_G relation. This change can be measured using the V_P versus V_G characteristic extraction technique presented in [2]. The non-uniform doping will change this characteristic, but it is best emphasized by looking at the variation of the gate voltage given by $V_G - V_{T0} - V_P$ which is nothing else than the ΔV_{TB} versus V_{ch} function given by (1) where V_{ch} is set to V_P. It is also interesting to evaluate the effect of non-uniform doping on the slope factor n as a function of the pinch-off voltage.

2. MODELLING THE NON-UNIFORM SUBSTRATE DOPING

The channel implant can be very precisely approximated by a Gaussian distribution. Taking such a profile into account would lead to a complicated expression of the threshold voltage V_{TB} [3][4]. Various semi-empiric or entirely empiric approaches have been suggested to model the threshold voltage of enhancement type devices with a single equation [5][6][7]; unfortunately, not all these models would work for a given technology.

Adequate solutions for circuit simulation generally require suitable approximations of the real doping profile, keeping a relation with the physical effects.

2.1. *The Step Model*

The real doping can be replaced by a step profile (doping N_s at the surface, and N_b in the bulk) [8] which leads to a simple expression of the threshold voltage, described by two parts, depending if the depletion depth W_m, related to V_{ch}, is smaller or larger than the implant depth W_i.

In VLSI devices, deep channel implants are needed such that the resulting implant depth is comparable to the depletion region depth in the back bias range of interest; consequently the step model becomes inaccurate when the channel voltage is such that W_m is around W_i. In addition, the discontinuity resulting from this profile is not ideal for circuit simulation because it could give rise to convergence problems.

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2.2. *Arora's Doping Transformation Model*

A similar approach has been used in [9], where the depletion charge is calculated for both cases $V_{ch} \le V_i$ and $V_{ch} > V_i$ using the same body effect factor than for the uniformly doped substrate case given by (5), but with the N_s term replaced by an effective concentration N_{eff}, which is a function of V_{ch}:

$$N_{eff} \equiv \begin{cases} N_s & \text{for: } V_{ch} \le V_i \\ N_{eq}(V_{ch}) & \text{for: } V_{ch} > V_i \end{cases} \tag{7}$$

$$N_{eq} \equiv N_s \cdot \frac{\Psi_0 + V_i}{\Psi_0 + V_{ch}} \cdot \left[1 - \frac{N_b}{N_s} + \frac{N_b}{N_s} \cdot \sqrt{1 + \frac{N_s}{N_b} \cdot \left(\frac{\Psi_0 + V_{ch}}{\Psi_0 + V_i} - 1\right)}\right] \tag{8}$$

V_i is the particular value of V_{ch} for which the depletion depth is equal to the implant depth:

$$V_i \equiv \frac{q \cdot N_s \cdot W_i^2}{2\varepsilon_s} - \Psi_0 \tag{9}$$

The variation of the threshold voltage ΔV_{TB} is expressed as:

$$\Delta V_{TB} = \gamma(N_{eff}) \cdot \sqrt{\Psi_0 + V_{ch}} - \gamma(N_s) \cdot \sqrt{\Psi_0} \tag{10}$$

This doping transformation model (Fig. 1) has very interesting features, but still requires two distinct sections.

2.3. *The MASTAR Model*

As for Arora's model, the MASTAR model [10] takes into account an effective concentration N_{eq}, function of the channel voltage. Arora's model is taken as a reference which is approximated by a single and continuous function (Fig. 1).

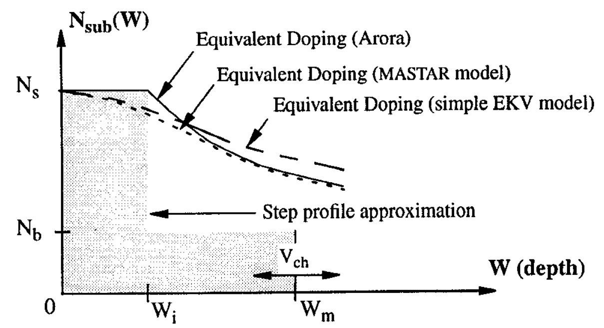

Fig. 1: *Modelling of the non-uniform doping of the substrate: Arora, Mastar and EKV (simple version) models*

A smooth transition between the doping at the surface and in the depth of the channel is obtained using a function *F*. The doping N_{eq} can be expressed as:

$$N_{eq} = N_s \cdot F^2 = N_s \cdot \left[\frac{f(V_{ch})}{f(0)}\right]^2 \tag{11}$$

with: $$f(V_{ch}) = \left[1 + \left(\frac{\Psi_0 + V_{ch}}{\Psi_0 + V_i}\right)^{3/2}\right]^{-1/2} + s \tag{12}$$

where: $$s = \sqrt{\frac{N_b}{N_s}} \tag{13}$$

The variation of the threshold voltage ΔV_{TB} is expressed as in (10), with N_{eff} changed to N_{eq} given by (11).

The parameters V_i and *s* are often found to differ significantly from their theoretical values and are thus to be regarded as fitting parameters.

2.4. *The simple EKV Model*

This solution consists in leaving GAMMA (γ) and PHI (Ψ_0) as two independent and constant parameters in the ΔV_{TB} definition in (10) where GAMMA $= \gamma(N_{eff}) = \gamma(N_s)$. This allows to take into account the non-uniform doping of the substrate without any additional parameters. This results in parameter values slightly smaller than theoretically, which permits to correctly describe the threshold voltage at small V_{ch}, without introducing large errors at strong V_{ch}. A simple and efficient method to obtain a pair of values of GAMMA and PHI satisfying this requirement is to extract these parameters directly from the measured V_P *versus* V_G characteristic [2]. This simple model can in fact be seen as an approximation of the effective doping obtained by the Arora model (Fig. 1). The advantage of this solution is its simplicity without additional parameters, maintaining an acceptable result for many realistic cases. Good results are achieved for p-channel devices without any further modifications.

2.5. *The improved EKV Model*

As for the MASTAR model, an approximation of the equivalent doping of the Arora model has been achieved by a simple and precise formulation using a single continuous function. The inverse of the equivalent doping of the normalized Arora model N_s/N_{eff} versus a normalized voltage $x = V_{ch}/V_i$ is nearly a linear function for $V_{ch} > V_i$ ($x > 1$). A simple interpolation function [11] is used to simultaneously describe the case $x < 1$ where the doping is supposed constant (Fig. 2).

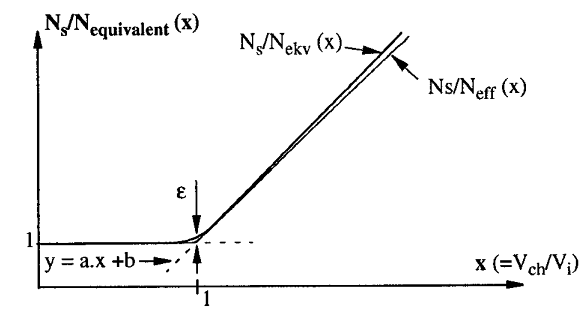

Fig. 2: *Modelling of the inverse (normalized) non-uniform doping: improved EKV model compared with Arora's model*

Finally, the non-uniform doping model can be expressed as:

$$N_{ekv}(V_{ch}) = \frac{N_s}{1 + \frac{1}{2} \cdot \left(G(V_{ch}) + \sqrt{G(V_{ch})^2 + 4 \cdot \varepsilon^2}\right)} \tag{14}$$

with: $$G(V_{ch}) = \alpha \cdot \left(\frac{V_{ch}}{V_i} - 1\right) \tag{15}$$

The parameter $\alpha \propto \sqrt{N_b/N_s}$ is adjusted by local optimization; ε is a fixed constant such that a smooth transition is obtained between the asymptotes $y = 1$ and $y = a.x+b$, around $x = 1$. The variation of the threshold voltage ΔV_{TB} is expressed as in (10), with N_{eff} replaced by N_{ekv}.

2.6. Discussion and comparison

For this simulation, we have defined a substrate doping by an ideal step, with $N_s = 2\times10^{16}$ cm^{-3}, $N_b = 7\times10^{14}$ cm^{-3}, $W_i = 0.4$ µm, and $T_{ox} = 20$ nm. The ΔV_{TB} *vs.* V_{ch} characteristic and the slope factor n *vs.* V_P are shown for all models in Fig. 3 and Fig. 4 respectively. Results for the improved EKV model and the Arora model are very close; the two other solutions differ but are nevertheless acceptable approximations, requiring more or less compromises. Note that the simple EKV solution is obtained without additional parameters. Table I shows the parameters used for this comparison.

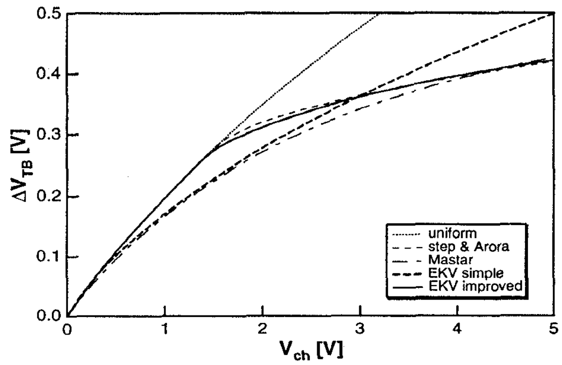

Fig. 3: *Simulation of ΔV_{TB} for all models*

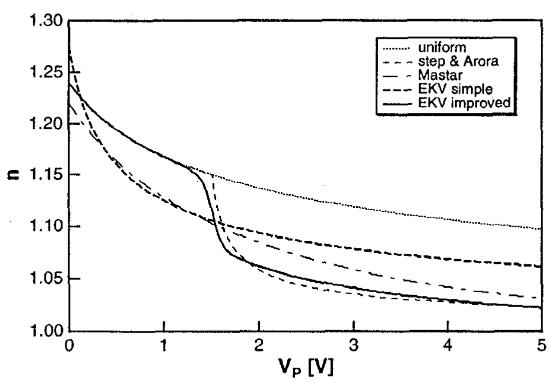

Fig. 4: *Simulation of the slope factor n for all models*

Table I: *Simulation parameters to model a step profile.*

	Ψ_0 (V)	PHI (V)	V_i (V)	s (-)	α (-)	$\gamma(N_s)$ (V$^{1/2}$)	$\gamma(N_b)$ (V$^{1/2}$)	GAMMA (V$^{1/2}$)
Step model	*0.96*	-	*1.5*	-	-	*0.47*	*0.08*	-
Arora's model	*0.96*	-	*1.5*	-	-	*0.47*	*0.08*	-
Mastar model	*0.96*	-	*4.7*	*0.23*	-	*0.47*	-	-
Simple EKV model	-	*0.26*	-	-	-	-	-	*0.28*
Improved EKV model	*0.96*	-	*1.5*	-	*0.3*	*0.47*	-	-

2.7. Comparison of the EKV solutions for a real doping profile

The 2-D simulator Medici was used to simulate a "measured" V_P vs. V_G characteristic, using the doping profile shown in Fig. 5.

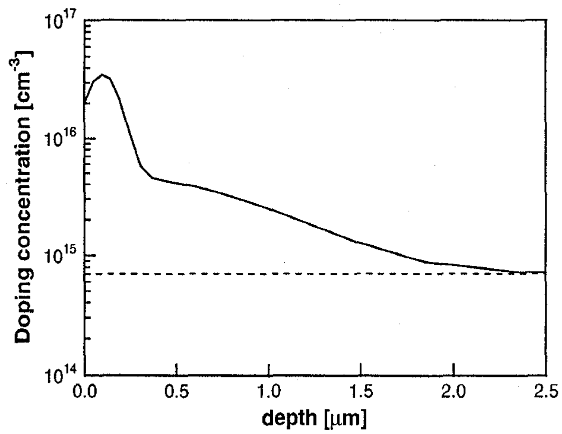

Fig. 5: *Substrate doping profile with a double implant used for the 2-D simulation*

Then, the respective parameters were determined for both EKV model solutions: V_{TO}, $\gamma(N_s)$, Ψ_0, $V_I{}^*$, and α for the improved model, and V_{TO}, GAMMA and PHI (leaving GAMMA and PHI as independent parameters) for the simple model, as shown in Table II.

Table II: *Parameter values for both EKV models for the simulated double implant profile*

Simple EKV Model			Improved EKV Model				
V_{TO}	GAMMA	PHI	V_{TO}	$\gamma(N_s)$	Ψ_0	α	$V_I{}^*$
V	V$^{1/2}$	V	V	V$^{1/2}$	V	-	V
0.75	*0.66*	*0.25*	*0.75*	*1.14*	*0.64*	*0.32*	*1.64*

The simulation of the V_P *vs.* V_G characteristic has been performed using (3) with the simple model, and the approximated expression of (16) with the improved model:

$$V_P(V_G) = V_G - V_{T0} + \gamma(Ns)\cdot\sqrt{\Psi_0} + \frac{\gamma'(N_{ekv})^2}{2} \qquad (16)$$

$$-\gamma'(N_{ekv})\cdot\left[\sqrt{V_G - V_{T0} + \Psi_0 + \gamma(Ns)\cdot\sqrt{\Psi_0} + \left(\frac{\gamma'(N_{ekv})}{2}\right)^2}\right]$$

where γ' is depending on V_G and V_I is replaced by $V_I{}^*$. The ΔV_{TB} *vs.* V_P simulated and "measured" characteristics and the slope factor n *vs.* V_P, are presented in Fig. 6 and Fig. 7 respectively. These results confirm the interesting features and the validity of the two models; one of which is precise and continuous, requiring two additional parameters, and the other is simple and computationally more efficient, without additional parameters, but giving a little less precise solution.

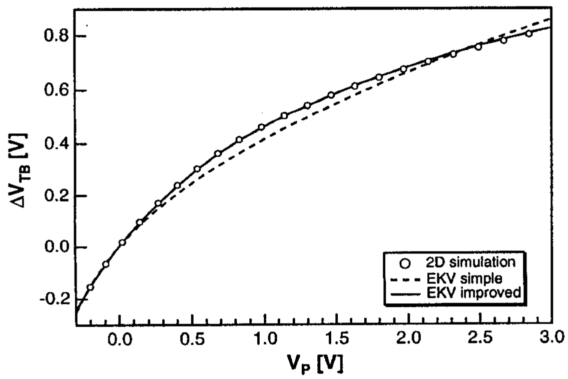

Fig. 6: *ΔV_{TB} versus V_P*

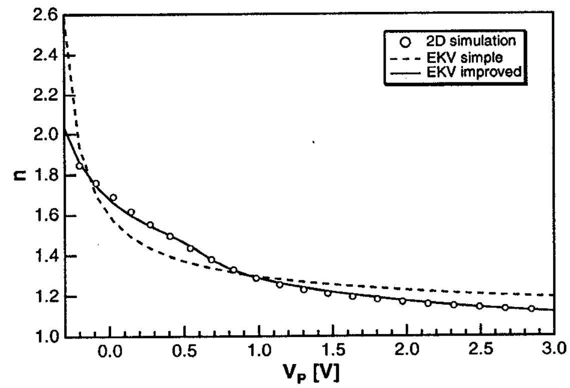

Fig. 7: *Slope factor n versus V_P*

2.8. Validation

The measured and simulated characteristics of ΔV_{TB} *vs.* V_P for an n-channel transistor of a 2 μm CMOS Low Voltage process presenting a strong non-uniform doping profile, are shown in Fig. 8.

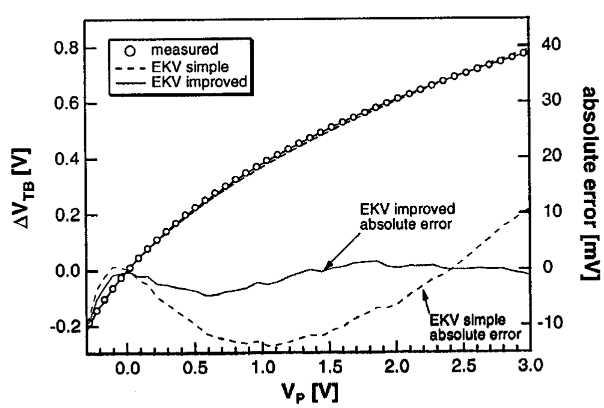

Fig. 8: *ΔV_{TB} vs. V_P characteristic (for a n-channel)*

For this real case, both proposed solutions give an excellent agreement between simulation and measured characteristics. The extracted parameters for both proposed solutions are found in Table III.

Table III: *Parameter values for both EKV models from the n-channel transistor of a 2 μm CMOS Low Voltage process*

Simple EKV Model			Improved EKV Model				
V_{TO}	GAMMA	PHI	V_{TO}	$\gamma(N_s)$	Ψ_0	α	V_I*
V	$V^{1/2}$	V	V	$V^{1/2}$	V	-	V
0.77	*0.64*	*0.36*	*0.77*	*0.75*	*0.46*	*89m*	*1.56*

3. CONCLUSION

Two new solutions for substrate non-uniform doping modeling have been discussed and compared with other approaches. Both proposed solutions are relatively simple and computationally efficient and can be used for MOST models where voltages are referred either to the substrate (EKV MOST model) or to the source (SPICE-like models). Validation on the ΔV_{TB} *vs.* V_P (ΔV_{TB} *vs.* V_{ch}) and the slope factor *n vs.* V_P characteristics show excellent agreement. The first model results from leaving both GAMMA and PHI as two independent parameters, leading to an advantageous compromise between simplicity and accuracy. The second model is a simple but precise and continuous solution, requiring only two additional parameters.

4. REFERENCES

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